Mathematical thaw

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For more than a decade, a significant number of new students have been lacking basic mathematical knowledge and skills as well as a conceptual understanding of mathematical content. At the same time, Abitur grades are getting better and better and the proportion of students who successfully complete the Abitur is increasing continuously. With the Bologna Process and the introduction of so-called educational standards, the government has mandated by law that German schools, universities and teacher training colleges use the language of competences as a universal language for describing, planning, testing and developing teaching and learning processes. This led to the removal of many classical topics from mathematics lessons. The shift to output orientation resulted in a break with the formerly internationally recognized educational tradition of the Enlightenment.

In the urgent reminder "from more than 130 professors and teachers" (Tagesspiegel 22 March 2017, Mathematikunterricht und Kompetenzorientierung - ein offener Brief [3]), the competence orientation in mathematics lessons is explicitly made responsible for the now undisputed decline in the elementary mathematical skills of a large number of students at the start of their university studies. The response from 50 members of our mathematical didactic guild is as follows (see [4]):

> There is no doubt that new students must have substantial basic mathematical knowledge (including the topics mentioned in the letter, such as fractions, binomial theorem, rearranging mathematical equations, elementary geometry or trigonometry). But they also have to be able to apply this knowledge in a meaningful way and use it to solve inner-mathematical problems as well as real-world problems; this is the basic idea of competence orientation in a nutshell.

What is the dispute about competence orientation all about? Is it not really a new word for something akin to learning goals? Has that not always been around? And who can object to schools and universities at least formulating the goal of ensuring that graduates are fully competent when leaving? Can anyone not want them to apply this knowledge meaningfully and use it to solve "inner-mathematical problems" as well as real-world problems? Do practitioners and theoreticians of mathematical doctrine focus on an enemy who is not actually suitable as such? Is it not good if the requirements are standardized so that they can be taught systematically? This critical discourse may appear as splitting hairs to large parts of a general mathematical audience.

The introduction of competence orientation as a universal and legally prescribed paradigm for the description and design of learning processes, however, has a very specific impact on the mathematical culture in teaching and research. It is vital for mathematics teachers in schools and universities to understand these developments more thoroughly and to engage in a humanities discourse with pragmatic consequences that cannot be fundamentally clarified by empiricism and that only partially takes place within the mathematical culture. In competence orientation, we are dealing with a fundamental change in our understanding of learning. Is it about learning to understand something; or is it about convincing others that they have understood something?

The conceptual system of competence orientation derives from applied psychology (Gelhard 2012). For a long time, it was used for the selection and adaptation of workers who are supposed to meet specially defined psychological requirements in the workplace, such as patience, accuracy, speed, etc. Although competence orientation with regard to teaching was promoted on the initiative of the OECD (see Weinert 2002, p. 27) primarily by pedagogical psychologists and educationalists working predominantly on a quantitative empirical basis, there is still no unequivocal empirical evidence to date that the competence orientation currently implemented by the state has a positive effect on the knowledge and skills of high school graduates or new students. Is it even possible to obtain such evidence? The problem is: The concept of competence is in principle not suitable for conceptually representing subject classification. This can only really be made clear if one looks at a specific subject matter. The *basic idea* outlined by the 50 educationalists is precisely not what constitutes competence orientation, but is simply yet another one of the promises of salvation that competence orientation likes to proffer. Using one of the disputes already conducted in the *Mitteilungen der DMV*, we will explain this in more detail.

Current rate of change of snow depth

In his contribution to the *Mitteilungen der DMV* (24-3, 2016), Franz Lemmermeyer explained the problems of the centralized Abitur examinations and the structure of current mathematics teaching in a well-founded way using a question from an Abitur paper from Baden-Württemberg as an example: The mathematical difficulty of the questions is predominantly middle-school level; where senior-level concepts are included in the tasks, the solutions are highly schematized and the function types and formulas are simplified special cases. The core curricula that have been gutted ("decluttered") of more complex content hardly permit the use of more complex formulas and functions. The frequently held view that the low mathematical complexity would be increased by being embedded in complex application contexts is a generalization that does not apply to the situation and that distracts from the concrete phony modeling.

In his letter to the editor (24-4, 2016), Werner Blum writes that he "does not want to generally defend the Abitur questions incriminated by Lemmermeyer". But he insists that solving these two questions requires an understanding "beyond mere reading requirements", such as comprehension of the "current rate of change in snow depth."

Let us take another look at the part of the question discussed by Lemmermeyer on the current rate of change in snow depth (A 2.1):

> The snow depth in a ski resort at a measuring point is 150 cm at 10:00 am. The current rate of change of this snow depth is described by the formula s with
>
> $s(t) = 16e^{(-0.5t)} - 14e^{(-t)} - 2;\ 0 \leq t \leq 12$
>
> *(t* in hours after 10:00 am, *s(t)* in centimeters per hour).

The *maximum local rate of change (a)* and an *integral-free function formula describing the snow depth at time t (b),* should then be determined respectively, before the local rate of change is changed once again by the use of snow cannons at a certain time in part (c) with a constant formula, and the snow depth should again be determined in the variegated situation.

This question is first of all about ski resorts and snow depths. Students picturing this context may now think of a snowy landscape with curved snow depths. This is reminiscent of function graphs. But watch out, there is a distractor lurking here: the question is about the dependency of snow depth on time. The attempt to understand this now recognized situation makes one think of a compacting snowdrift onto which more snow falls from time to time. What does *current rate of change* mean here in the context of melting and accumulating snow crystals? What exactly is changing here and what does *currently* mean? And why should one be able to measure the current rate of change more easily than the snow depth itself? This is all not easy.

> An initial, probably naive attempt to create hypotheses on this such as "if it is snowing consistently, the local rate of change is constant" or similar however, would not be wise: not because the situation is actually more complicated, but because this would only create further distance from the successful completion of the Abitur question. Now the content of prior lessons proves advantageous: Read "current rate of change" and understand "derivative". This current rate of change of the snow depth is described by the function s with $s(t) = 16e^{(-0.5t)} - 14e^{(-t)} - 2;\ 0 \leq t \leq 12$. Insiders know that (a) this is not the case, and (b) this is because, for lack of further mathematical skills, the students may only be confronted with polynomials or linear combinations of equations of the type *e-function exponentiated by linear formula*. The latter are so-called D-functions (Carl-Peter Fitting), since in North Rhine-Westphalia such developments always come from Düsseldorf. Incidentally, in 2007, prime number, prime factorization, altitude and cathetus theorem, sine and cosine rule, logarithm laws, and much more, and then in 2014 the quotient rule, the broken-rational functions, and the cosine function, were "cleared out".

With extensive practice, students are still able to differentiate (and integrate) D-functions by writing the exponent in front of exponential functions and decreasing them by 1, and for *e-function exponentiated by linear function,* simply writing the increase of the linear function in front. That is not too difficult to remember. Incidentally, this hegemonic knowledge of differentiation and integration of D-functions may prevent ninth-graders from already being able to solve the Abitur question, as has long been the case in biology (Jahnke & Klein, 2012). For Austria, Bandelt and Kühnei (2016, 2017) have shown that the mathematical leavers' examination can even be passed with the material taught in years 9 and 10, and even with the grade *satisfactory.*

Competences - a tool of applied psychology

The question discussed by Lemmermeyer is an example. It is important to understand that the imperatives used do not in any way demand what they imply or what the situation would dictate, but that they are so-called *operators*, as we will explain in more detail below. In this way, and through the contexts that appear application-related, the questions seem demanding and can withstand the first glance of educational economists, parents, politicians (and perhaps even some mathematically educated members of the university). However, these are standardized types of questions that are always worked through according to the same schemes practiced dozens of times (see Jahnke 2016). The cheap, competence-oriented formulations of the education standards do not provide any orientation regarding the content.

In fact, it is the central examinations and not the core syllabi that have become a secret syllabus, and thus, incidentally, they are no longer the subject of democratic decision-making. Introduced under the pretext of replacing the culture of fixed solution schemes, it is precisely these solution schemes that have become the central teaching subject under competence orientation. The real difficulty of the questions is not so much mathematics per se, but rather freeing the mathematical problems from their textual distractors and finding out which of the practiced schemes must be activated and imitated. The (high school) math class is in crisis!

The root of this development is seen by more and more subject educationalists and pedagogues to lie in competence orientation. It penetrates deeper and deeper into all areas of the German education system. At the instigation of the German Rectors' Conference, it will take even more radical forms at universities than it has already been doing since the introduction of the Bachelor-Master system. The goal of politics is to improve the result of tests such as PISA or TIMSS. Competence orientation promises, among other things, to be able to do this and turn education into a manageable system:

> Educational standards with their reference to student competences are explicitly formulated in a way that allows them to be checked with the help of corresponding questions or tests. This measurability characterizes them nationally and internationally, and with all due modesty, it is this characteristic that makes it possible to determine at certain points in time whether and to what extent students are adequately prepared for life or whether there is a need for optimization. [Blum et al. 2006, p. 9]

Competence orientation in its economic language corresponds to the requirements of the Organization for Economic Co-operation and Development (OECD) for both education and individuals to be interpreted in economic terms as *human capital*. Educational scientists such as Franz Weinert (2002, p. 26) and many others went on to implement this.

In the question shown, the students were required to demonstrate modeling competence. As we see here, questions are never directly about content, as content simply cannot be formulated in this language. The frequently mentioned competences of being able "to prove" or "to argue" (Blum 2016) concern text-based questions and their text-based solutions that duly contain the key words expected by the people who wrote the questions.

"You should knit with wool" (Blum 2016). But it is never said which wool.
It also does not become clear which wool is the best wool for knitting. Content is only treated as a paradigm and not as exemplary: the latter would require an understanding of the role of content in the subject culture.

How can one develop a number concept without prime numbers, geometric perspective without elementary geometry, real numbers without irrationality, trigonometry without unit circle and cosine function, scalar product without the law of cosines, analytic geometry without geometry, linear algebra without linearity, combinatorics without counting method, binomial distribution without the binomial theorem, differential and integral calculus without a (only suitable for demonstration) limit value concept, mathematical applications without applicable mathematics, mathematical modeling without mathematical models, mathematical thinking without explanations and evidence?

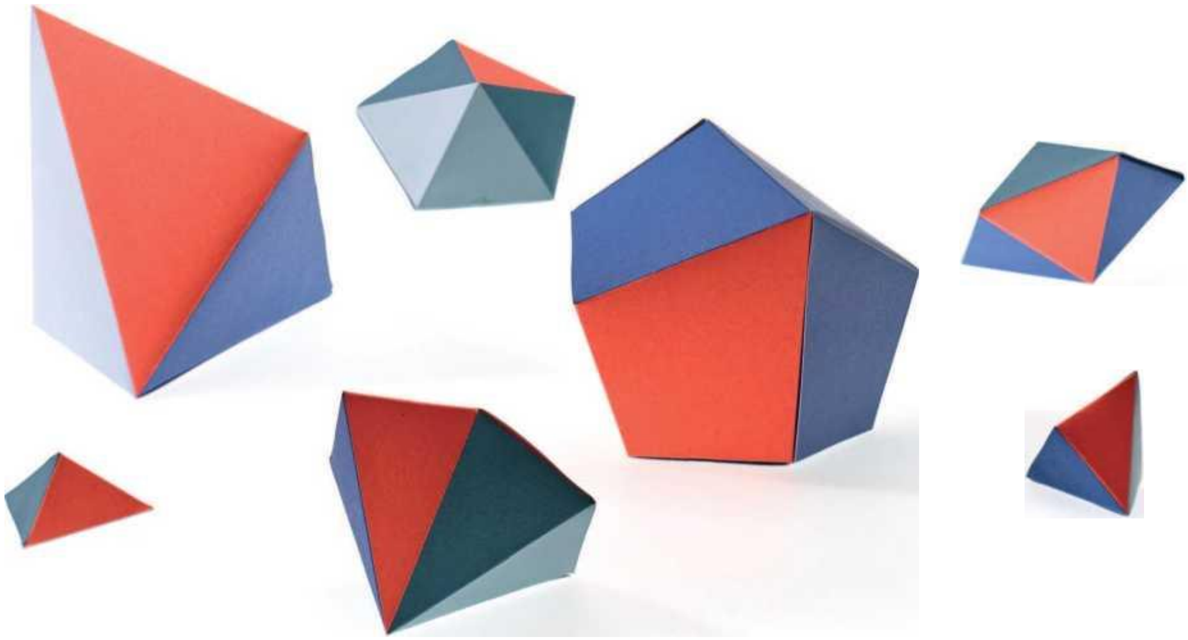
Competence orientation currently makes all this possible in Germany! After all, education is no longer about understanding content, but about acquiring competences and achieving *revenues from school learning* (Weinert 2002, p. 28).

> In this context, the OECD has repeatedly suggested that the ambiguous concept of performance should generally be replaced with the concept of competence (see [...]). Competences in this context are the cognitive abilities and skills available to or learnable by individuals in order to solve specific problems and the associated motivational, volitional and social readiness and ability to successfully and responsibly use the solutions in variable situations. (Weinert 2002, p. 27-28)

The concept of competence based on this definition by Franz Weinert, the credo of the testing industry, became the central concept of the transformation of our entire education system. It has evolved from a psychological selection tool into the guiding principle for industrial quality control of human capital suppliers of economic systems, as the OECD has been doing on a regular basis for decades.

After all, and this cannot be emphasized enough, competences are a psychological instrument. *Modeling, collaborating, arguing* and even *moral competences* (Weinert 2002, p. 28), etc. are elevated to context-free problem-solving activities. As they do not have to do justice to any context, they become observable and measurable psychological categories.

For example, modeling is performed when using the vocabulary of a didactic modeling cycle. In addition to the psychological selection and adaptation function, the competences have an additional background, which is already hinted at in Weinert *(2002,* p. 18) when, in the name of the "performance-oriented educators and citizens", he identifies performance as a "manifestation of a basic human need, an opportunity of individual self-realization through the experience of self-efficacy". Weinert considers intrinsic motivation to be one of the factors whose "effectiveness is still overestimated in some of the literature" *(2002,* p. 25). As early as the 1980s, the psychologists Deci and Ryan (2000) developed a "self-determination theory" that was widely adopted in teaching methodology and describes the internalization of extrinsic motivation through the experience of *competence, autonomy* and *social integration*. The output orientation brings together the selection and the motivational dimension of competence by using the operationalizations (see Baumert et al 2000, p. 11) of such motivation theories as proof of the "motivational preferences" of the students as part of a general concept of competence. These motivational preferences are consequently also gathered within the PISA study (Baumert et al., 2000).



(Illustration: Christoph Eyrich)

It may be pointedly said: The state-mandated competence orientation is a method that strives to make human capital usable for the promotion of economic development through the internalization of extrinsic motivations (see Weinert, p. 26), which, as we now know in mathematics, does not even work.

In the tradition of education, which began with Plato, and with the Enlightenment and the New Humanism went on to become the basis of the European understanding of education, it is the objects, phenomena and spiritual creations of this world itself that fascinate us with their Eros (Plato) and induce us to search for the true, the good and the beautiful (Dörpinghaus 2009). Kant's maxim *Sapere aude!* calls for thinking on the basis of one's own motivation. The psychological instrument of competence, on the other hand, achieves its effect simply by being subjectively experienced, thus providing an emotional basis for the subsequent external treatment, without necessarily having to do justice to an object, a phenomenon or a spiritual creation.

In our example, it is enough to feel that you can say something meaningful about snow depths. Competence can therefore, seemingly across all subjects, be used as a definition for universal skills that do not require any subject-specific foundations. These universal skills can be used to imitate the important skills established in the individual specialist cultures, as long as they are perceived by the testers as competence. Corresponding concepts and their understanding, as well as the proof that this understanding is in place, are taken out of context and operationalized through psychological tests. This then produces the strangest subject-specific consequences. A well-founded and wittily-written read on these consequences is (Klein 2016).

Operators as totalitarian language rules

Even in elementary mathematics, teachers and students, for example, wonder whether an argumentation can be considered as proof, or whether a calculation leads to a logically reliable statement as to whether a drawing reflects a mathematical situation, if and to what extent a derivation can be considered successful, or which property of an object should be elevated to a definition. Mathematical development takes place through discourse on these and similar questions. However, in the course of this, mathematical terms are surprisingly characterized by an intrinsic or Platonic inevitability. Nonetheless, here and there it also makes sense to discuss the introduction of terminology. Should plane polygons also include digons? From the point of view of the inner angle sum, yes, from the point of view of triangulation ability however, no. In such open discourses, mathematics differs, for example, from jurisprudence which is based on established laws.

Just as Facebook has redefined the term *friend*, the competence apologists and their operators have for many years now been taking away people's own language with its open, creative possibilities in mathematics, without most of them being aware of this, let alone being upset about it. This must be known if one wants to understand the often demanding-sounding wordings of the Abitur questions. The verbs *specify, name, compute, describe, create, represent, sketch, draw, plot, identify, determine, decide, explain, deduce, interpret, investigate, verify, compare, show, prove, conclude, demonstrate* and *disprove* have always been understood in mathematics in the way that has been experienced as meaningful in the dialogue between mathematicians, teachers and students. Mathematical ability was particularly demonstrated by such understanding, with the struggle for an interpretation also resulting in development and interesting insights.

Now the operators are fixed as unequivocal instructions, as commands. In most of the federal states (for North Rhine-Westphalia see [1]), the state, within the framework of the so-called *safeguarding of standards* (What exactly is being safeguarded against what?), has legally specified the definitions of each of the above verbs in three requirement areas. This mathematical jurisprudence not infrequently results in nonsense. Why do we think so? Well, we can judge that; just take a look ...

> Formulating and justifying an independent verdict, using specialist knowledge and methods.

Looking at the operator *specify, name*, we find:

> Listing objects, facts, concepts, data without further explanation, justifications and without presenting solution approaches or procedures.

That sounds harmless and simple, like the famous (and apparently no longer applicable) central service provision of the Bundeswehr ZDv 10/5 Life in the Field: "From a water depth of 1.20 m onwards, the soldier has to independently commence swimming motions. The saluting duty is omitted." But the operators are not just curious formulations of the obvious; they are also dangerous, because they absolve the learner from thinking about the meaningfulness of his answer. The operator *determine* appears innocent with: *Identifying contexts or solutions, presenting the procedure and formulating the results.* Yet in practice this often results in nothing other than *teaching reduced to practicing the translation of modeling tasks into calculator instructions*, much criticized by Lemmermeyer. The situation is similar with other operators.

Even in university teacher training, and certainly also in other courses of study, students are increasingly demanding the use of these state-mandated operators ("beautifully clear imperatives", quote from a student), with which one no longer has to worry about whether or not the respective answer is meaningful and satisfactory in context. If you formulate in an exam, for example, that the students should specify something, then one must not be surprised if the students list "objects, facts, concepts, data without further explanation, justifications and without presenting solution approaches or procedures", even if this appears absurd in the respective situation. In the exam review, the reference follows that the task is handled correctly by listing a few keywords because the operator *specify* is fulfilled. Are we the only colleagues who experience this sort of thing?

Modeling and application

In their reaction to Lemmermeyer's critical analysis (Greefrath et al., 2016), Gilbert Greefrath, Matthias Ludwig and Stephan Silier argue that during reading, the impression emerges "... that mathematics is not understood as a general education subject here, but one-sidedly as an inner-mathematical one". They explain the meaning of "general knowledge" based on Winter's Fundamental Experiences. But who could actually be against Winter's Fundamental Experiences? This is not where the problem lies. They further state in their reply: "All three fundamental experiences are equally important and also form the basis for current curricula and educational standards." This however is wrong.

For example, the state government in North Rhine-Westphalia has only seemingly adopted these fundamental experiences without further comment; they were *competencified.* In the curriculum navigator SII Gymnasiale Oberstufe Mathematik [Senior Level Mathematics] [2] we find as introduction to the core curriculum and the tasks and objectives of the subject, something that at first glance looks like Winter's Fundamental Experiences, but on closer inspection means something completely different.

For example, *perceive* and *understand* is different from *perceive, understand, assess and influence.* Can and should students assess and influence technical, natural, social and cultural phenomena and processes? Does this not require you to have legitimacy, experience and competence in the true sense of the word? The fact that the state government wants to have the students develop mathematical objects and facts that were previously recognized to be a pattern and not learned and understood as mathematics, is only comprehensible to those who are used to such grandiose prose coming from education economists.

Most importantly however, the problem-solving ability described by Winter refers to the culture of solving problems in mathematics, such as those described by Polya, and to modeling in real contexts. The problem-solving ability in the core curriculum, however, refers to one of the most controversial concepts of competence orientation (see for example Wiechmann 2013 or Gelhard 2012). It has virtually nothing to do with solving mathematically demanding questions and problems, as we have seen not only in Abitur exams for many years. Consequently, these competence-oriented problem-solving skills should not just be available, but also usable, and usually not by the learner himself. The latter, however, should be willing to be used.

From a competence perspective, problem-solving skills are often associated with the application of mathematics and modeling with mathematics. Both had an important, but not universal, significance for mathematics education before the PISA shock. In the past, numerous subjects, not least physics and many other now demathematized subjects were responsible for the application of mathematics: Computer science, chemistry, biology, geography, economics, general studies, ...

But even in German mathematics lessons, *apply* and *model* have a long tradition, and many successful examples have been developed over the centuries. These also include contemporary motivating contexts, and among those there are also some by the colleagues Blum, Greefrath, Ludwig and Silier. But real application, and especially modeling, are hard. They are only mastered for particularly dedicated teachers with scientific relevance to the contexts. Nevertheless, modeling is required in core curricula; the

Fundamental experience after Heinrich Winter

(G1) "perceiving in a specific way and understanding manifestations of the world around us that affect or should affect us all, from nature, society and culture", (G2) "getting to know and understanding mathematical objects and situations, represented in language, symbols, pictures and formulae, as mental creations, as a deductively ordered world of their own kind, and" (G3) "acquiring problem-solving skills that go beyond mathematics (heuristic abilities) by dealing with questions."	Students should ... perceive, understand, assess and influence technical, natural, social and cultural phenomena and processes with the help of mathematics (mathematics as an application). ... recognize and further develop mathematical objects and situations, represented in language, symbols and pictures, as a deductively ordered world of their own kind (mathematics as a structure), ... in dealing with mathematical questions, acquire and use creativity and problem-solving skills that go beyond mathematics (mathematics as an individual and creative activity).
Core curriculum NRW	

certainly meaningful classical testing of the knowledge of mathematical terms in metaphorical contexts is considered backward.

In their letter to the editor, the three authors also distance themselves from the Abitur question detailed above as an example for modeling tasks by citing the currently common mathematical-didactic features and classifications for good modeling tasks. They generally regard the consideration of Abitur questions unsuitable for the analysis of mathematics education, but do not make it clear that modeling tasks are not suitable at all for Abitur questions and other centralized examinations.

The issue is not modeling questions, but pseudo-modeling questions such as those usually found in Abitur exams. The reason for the popularity of the latter, and thus also for the popularity of modeling mathematics didactics, lies in the significance of these problems for the pretense of a non-existent level and the imitation of complex mathematical activities, as promoted by PISA and other output research. Because pseudo-modeling problems do not derive their relevance for school from multidisciplinary approaches and thus from a conscious mathematization of scientific topics, but from the pretense of universal modeling skills that do not have to be developed as an example in application contexts with a considerable amount of time and effort. In order to maintain this absurd practice, the subject-related discussion about the chimera of generally accepted universal competences is ideologized and only guided by psychological instruments.

The crisis is manifest

The mathematical deficits of university students can no longer be compensated with bridging courses and preliminary courses. It could be bluntly put that, following the *New Math* movement with its anti-didactic inversion (Freudenthal) since the turn of the millennium, the consequences of which are still felt today, the orientation towards competence and output has led to a *No Math* movement marked by anti-didactic omission. If school mathematics education continues to evolve into a parallel culture that becomes more and more detached from the ancient culture of mathematics, students of mathematics or related subjects will not only become mathematically weaker, but they will be absent, as we saw in an extreme form in the Netherlands approximately ten years ago (Kaenders 2009). Something urgently needs to happen if we want to limit the damage to the mathematical culture.

In its statement [4] "On the Current Discussion on the Quality of Mathematics Education" of April 20, 2017, the Mathematics Commission *School-University Transition* of the associations DMV, GDM and MNU recognizes the crisis of mathematics education for the first time in clear terms:

> For more than a decade, German universities have been alarmed to note that a significant number of new students lack basic mathematical knowledge and skills as well as a conceptual understanding of mathematical content.

The Commission blames this on the reduction of mathematics education in the different time tables of the federal states, the elimination or change of the Leistungskurs [advanced course] system, the high proportion of Abitur graduates in each year, the conflict between general education and the preparation for university mathematics courses and the introduction of new content. Since its inception, the Commission has taken on the important task of calling on policymakers to improve these frameworks [5].

However, on behalf of the professional associations, the Commission also recommends continuing, and even intensifying, the focus on competence-based education standards. It is *undeniable that the path taken with educational standards is the right one in terms of education policy and that it makes it possible to better harmonize the provisions that until now have varied so much across Germany. They should now be made even more binding and worded more precisely.*

We believe that it is very important for this conclusion by the Commission to be first of all considered in a well-founded theoretical discourse before the proposed path of competence orientation is pursued. Otherwise, if the Commission's appeal to policymakers on the framework for mathematics education is heard, this means: In the future, snow depths will be modelled with D-functions throughout Germany even more intensively than before in preparation for a mathematics-based degree in more hours per week, partly in advanced courses, with slightly fewer high school students than now, and the quality of the whole thing is assured by IQB psychologists.

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[3] https://tinyurl.com/yd7pvvpr (04/24/2017)
[4] https://tinyurl.com/yaslmdzf (04/24/2017)
[5] https://tinyurl.com/yc3y4u94 (04/24/2017)

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